

## Dual-Mode Dielectric-Loaded Resonators With Cross-Coupling Flats

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### Abstract

Cross coupling in dual-mode dielectric resonators is obtained by introducing asymmetries in the normally circular cross section of the resonator in the form of flattened regions at an angle of 45 degree to the two orthogonal modes. When the dielectric resonator occupies the full length of the cavity the coupling is obtained based on the relationship between the waveguide polarizer and filter coupling coefficient as described in [1], and a perturbation theory is applied to obtain the dimensions. The theory has been checked against results obtained by a three dimensional field theory program. When the dielectric resonator is shorter than the cavity the simple relationship between waveguide polarizability and filter coupling coefficient no longer applies, but good results are still obtained from perturbation theory.

### I. Introduction

A dual-mode dielectric loaded cavity is shown in Fig. 1. Cross coupling between the modes is normally effected by a tuning screw at a 45 degree angle to the orthogonal modes. Since most of the field lies within the dielectric the screws tends to be quite close to the dielectric for sufficient coupling to take place. This may cause dimensional problems and difficulties with temperature compensation between the aluminum housing and the ceramic resonators.

An effective solution to these problems was the realization of the waveguide polarizer by flattening the opposite sides of the resonator as shown in Fig. 1(b). The flattening required is very slight, and therefore the change of the cut off wavelengths for the parallel and perpendicular fields may be calculated using perturbation theory similar to [1].

When the dielectric resonator occupies the entire length of the cavity, the relationship between waveguide polarization and coupling coefficient given in [1] applies, except that the constant of proportionality multiplying the volume displaced by the flats, e.g. 0.225 in [1, Equ. (40)], is now frequency dependent due to dispersion of the inho-

mogeneous dielectric waveguide.

In the more usual case where the dielectric is shorter than the metallic cavity the situation is more complicated than in the full length case [2]. However perturbation theory is still applicable, although no simple relationship exists between dielectric waveguide polarization and coupling coefficient.

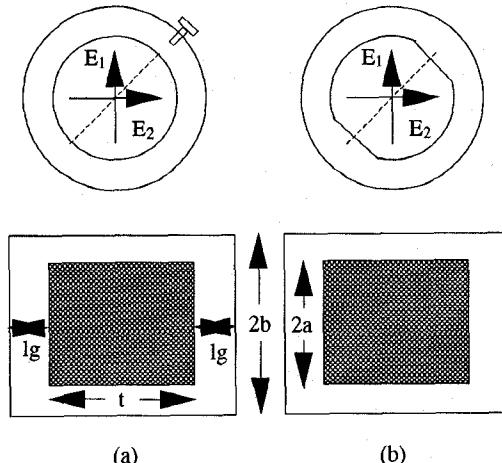


Fig. 1 Dielectric-Loaded Resonator with (a) cross-coupling tuning screw, and (b) cross-coupling incorporating 45° flats.

### II. Perturbation Theory of Dielectric Materials in Cavities

Instead of perturbation of the cavity boundary, the dielectric which forms the flats is changed from high dielectric constant to one for dielectric loaded waveguides and cavities. The change in resonant frequency by perturbation of the materials in cavity is [3]:

$$\begin{aligned}
\frac{f-f_o}{f_o} &= \frac{\Delta f}{f_o} \\
&= -\frac{\iiint_{\tau} (\Delta \epsilon \mathbf{E} \cdot \mathbf{E}_o^* + \Delta \mu \mathbf{H} \cdot \mathbf{H}_o^*) d\tau}{\iiint_{\tau} (\epsilon \mathbf{E} \cdot \mathbf{E}_o^* + \mu \mathbf{H} \cdot \mathbf{H}_o^*) d\tau}
\end{aligned} \tag{1}$$

where  $\mathbf{E}_o$ ,  $\mathbf{H}_o$  are the fields,  $f_o$  is the resonant frequency of the unperturbed cavity,  $\mathbf{E}$ ,  $\mathbf{H}$  and  $f$  are those after perturbation.  $\tau$  is the volume of the cavity.

For the case of dielectric resonator loaded cavity, only small portion of the dielectric material is removed to form the flats, the denominator of the right hand side in (1) can be computed approximately as:

$$\begin{aligned}
&\iiint_{\tau} (\epsilon \mathbf{E} \cdot \mathbf{E}_o^* + \mu \mathbf{H} \cdot \mathbf{H}_o^*) d\tau \\
&\approx \iiint_{\tau} (\epsilon \mathbf{E}_o \cdot \mathbf{E}_o^* + \mu \mathbf{H}_o \cdot \mathbf{H}_o^*) d\tau
\end{aligned} \tag{2}$$

which is proportional to the total energy of the cavity.  $\Delta \mu$  is zero for the dielectric loaded cavity and (1) is simplified as:

$$\frac{\Delta f}{f_o} \approx -\frac{\iiint_{\tau} (\Delta \epsilon \mathbf{E} \cdot \mathbf{E}_o^*) d\tau}{\iiint_{\tau} (\epsilon \mathbf{E}_o \cdot \mathbf{E}_o^* + \mu \mathbf{H}_o \cdot \mathbf{H}_o^*) d\tau} \tag{3}$$

### III. Full-Length Dielectric-Loaded Cavities

For the case of the full-length dielectric-loaded cavity, since most of the stored energy lies within the dielectric and the electric energy is equal to the magnetic energy, (3) can be rewritten as:

$$\frac{\Delta f}{f_o} \approx -\frac{\iiint_{\tau} (\Delta \epsilon \mathbf{E} \cdot \mathbf{E}_o^*) d\tau}{2 \iiint_{\tau} (\epsilon \mathbf{E}_o \cdot \mathbf{E}_o^*) d\tau} \tag{4}$$

Only the electric fields of the  $HE_{11}$  mode within the

dielectric are required to compute (3) and they are readily derived in [2], [4]:

$$E_z = AJ_1(\xi_1 r) \cos\theta \tag{5}$$

$$E_r = -\frac{A\gamma}{\xi_1^2} \left[ \xi_1 J_1'(\xi_1 r) + \frac{\alpha J_1(\xi_1 r)}{r} \right] \cos\theta \tag{6}$$

$$E_\phi = \frac{A\gamma}{\xi_1^2} \left[ \frac{J_1(\xi_1 r)}{r} + \alpha \xi_1 J_1'(\xi_1 r) \right] \sin\theta \tag{7}$$

where  $A$  is an arbitrary constant,  $\gamma$  is the propagating constant and  $\xi_1^2 = \epsilon_r k_o^2 + \gamma^2$ .

In order to compute (5), (6), (7),  $\gamma$ ,  $\alpha$  and  $\xi$  should be solved. The propagating constant  $\gamma$  must be computed numerically by solving a characteristic equation.  $\alpha$  is an analytical function of  $(\alpha, \xi_1, \zeta_2)$ , where  $\zeta_2^2 = -(k_o^2 + \gamma^2)$ . The integration of the denominator in (3) can be rearranged as:

$$\begin{aligned}
U_T &= 2 \iiint_{\tau} (\epsilon \mathbf{E}_o \cdot \mathbf{E}_o^*) d\tau \\
&= 2\pi \int_{-\pi}^{\pi} \int_0^a (\epsilon \mathbf{E}_o \cdot \mathbf{E}_o^*) r dr d\theta \\
&= A^2 \pi^2 \epsilon \int_0^a J_1^2(\xi_1 r) r dr \\
&+ \frac{A^2 \gamma^2 \pi^2 \epsilon}{\xi_1^4} \left\{ (1 + \alpha^2) \int_0^a \left( \frac{J_1^2(\xi_1 r)}{r^2} + \xi_1^2 J_1^2(\xi_1 r) \right) r dr \right. \\
&\left. + 2(1 + \alpha) \int_0^a \xi_1 J_1(\xi_1 r) J_1'(\xi_1 r) r dr \right\}
\end{aligned} \tag{8}$$

If the perturbed fields are assumed to be uniform and equal to the unperturbed fields at  $r = a'$  ( $a'$  is somewhere between  $a$  and  $a-t$ ), the nominator in (3) can be re-written as:

$$\begin{aligned}
&\iiint_{\tau} (\Delta \epsilon \mathbf{E} \cdot \mathbf{E}_o^*) d\tau \\
&= \Delta U_A \int \cos^2 \theta d\theta + \Delta U_B \int \sin^2 \theta d\theta
\end{aligned} \tag{9}$$

where

$$\Delta U_A = \Delta \epsilon \frac{A^2 \gamma^2}{\xi_1^4} \left\{ \left[ \xi_1 J_1(\xi_1 a') + \frac{\alpha J_1(\xi_1 a')}{a'} \right]^2 + J_1^2(\xi_1 a') \right\} \quad (10)$$

$$\Delta U_B = \Delta \epsilon \frac{A^2 \gamma^2}{\xi_1^4} \left[ \frac{J_1(\xi_1 a')}{a'} + \alpha \xi_1 J_1(\xi_1 a') \right]^2 \quad (11)$$

Denote  $K_A = \Delta U_A / U_T$  and  $K_B = \Delta U_B / U_T$ , the results for the parallel and perpendicular  $HE_{11}$  modes, as shown in Fig. 2, are:

$$\Delta f_{\parallel}/f_o = K_A \int \sin^2 d\theta + K_B \int \cos^2 d\theta \quad (12)$$

$$\Delta f_{\perp}/f_o = K_A \int \cos^2 d\theta + K_B \int \sin^2 d\theta \quad (13)$$

Neglecting the term containing  $\int \sin^2 d\theta$  since

that flat is near  $\theta = 0$ , one has:

$$\Delta f_{\parallel}/f_o = K_B \left[ \frac{1}{2} \sin 2\phi - \phi \cos 2\phi \right] \quad (14)$$

$$\Delta f_{\perp}/f_o = K_A \left[ \frac{1}{2} \sin 2\phi - \phi \cos 2\phi \right] \quad (15)$$

where  $\phi = \cos^{-1}(1-t/a)$ .

#### IV. Results

##### Full-Length Dielectric-Loaded Cavities

In order to verify our theory, the results is checked by a three dimensional field theory program-Hewlett-Packard HFSS program. The HFSS is applied to

solve the propagating constant of the dielectric-loaded waveguide ( $\beta$ ) with unperturbed and perturbed dielectric. The shifting of the propagating constant then is transferred to the shifting of the full-length dielectric cavity resonant frequency by:

$$\frac{\Delta f}{f_o} \equiv \frac{d\beta}{\beta} \left( \frac{1}{1 + k_C^2/\beta^2} \right) \quad (16)$$

where  $k_C$  is defined by  $\beta^2 = \epsilon_r k_o^2 - k_c^2$ .  $k_C$  is frequency dependent but is almost constant when the frequency is far away from the cutoff frequency. The results of HFSS for a dielectric constant of 37.3 dielectric-loaded waveguide and inner diameter of 0.760", outer diameter of 1.206" are listed in Table 1. The results derived from (14), (15) are shown in Fig. 3, where  $a'$  in (10), (11) is chosen as  $(a-t/2)$ . They agree well with the HFSS results and perturbation theory is verified.

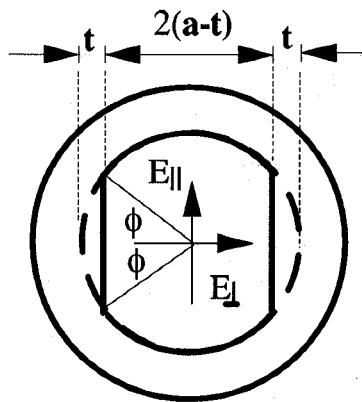


Fig. 2 Cross section of dielectric-Loaded Waveguide

##### Partial-Length Dielectric-Loaded Cavities

For a partial-filled dielectric loaded cavity, as shown in Fig. 1, more than one propagating mode may be

Table 1 The results of HFSS for a dielectric loaded waveguide

Freq. (GHz)	$E_{\perp}$ mode							
	$\Delta\beta/\beta$	$t' = 0.0394$	$t' = 0.0789$	$t' = 0.1184$	$\Delta f/f$	$t' = 0.0394$	$t' = 0.0789$	$t' = 0.1184$
3.75	0.00180	0.005910	0.01165	0.00139	0.00458	0.00904		
3.00	0.00375	0.01179	0.02300	0.00239	0.00753	0.01469		
2.50	0.00689	0.00237	0.04758	0.00327	0.01130	0.02260		
Freq. (GHz)	$E_{\parallel}$ mode			$\Delta f/f$				
	$\Delta\beta/\beta$	$t' = 0.0394$	$t' = 0.0789$	$t' = 0.1184$	$t' = 0.0394$	$t' = 0.0789$	$t' = 0.1184$	
3.75	0.00050	0.00182	0.00384	0.00038	0.00140	0.00295		
3.00	0.00083	0.00256	0.00531	0.00053	0.00164	0.00340		
2.50	0.00016	0.00227	0.00590	0.00008	0.00109	0.00283		
$t' = t/a$								

excited in the dielectric-loaded waveguide region. Simple formulation as in the case of the full-length dielectric loaded cavity is invalid. Mode-matching technique [5], [6] can be used to solve the unperturbed resonant frequency and field distribution rigorously.

Fig. 4 shows the results for a partial-length dielectric-loaded cavity. The shifting of the resonant frequency by flattening the DR is a strong function of the distance between DR and the end plates ( $l_g$ ). Since more than one propagating modes are excited by the discontinuities, the field distribution within the DR depends on the energy distribution among them.

## V. Conclusions

Perturbation theory has been applied to compute the coupling between dual-modes dielectric-resonators by means of flats surfaces at 45° locations relative to the polarization of each mode. This method can be used to avoid metallic tuning and coupling screws which degrade the unloaded Q's and reduce the power handling capability.

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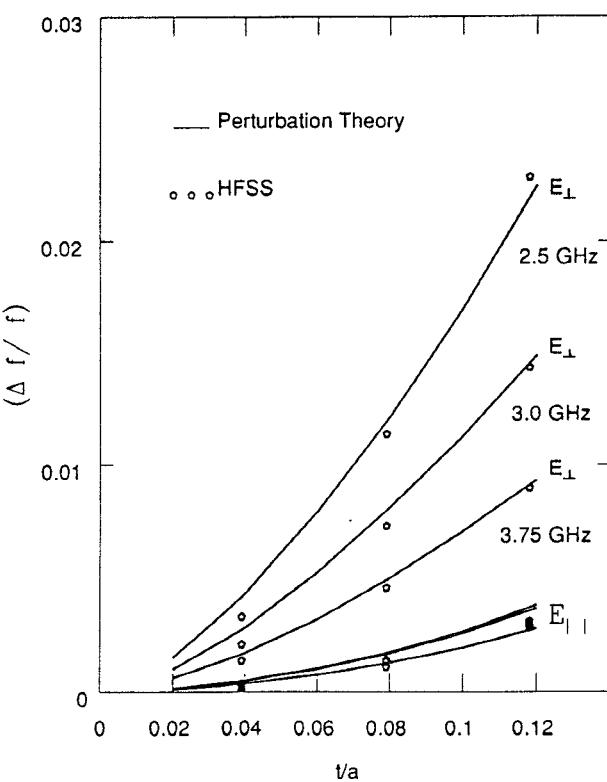


Fig. 3 The compared results of perturbation theory and HFSS.

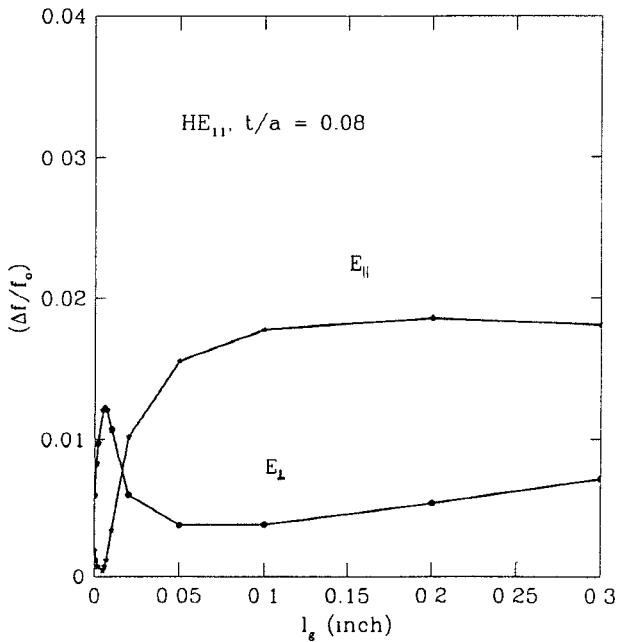


Fig. 4  $(\Delta f / f_o)$  of a partial-length dielectric-loaded cavity